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# Statistical error reduction for correlation-driven operational modal analysis

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## Abstract

Statistical errors effect the estimated correlation function matrix in Operational Modal Analysis due to the finite time length of the sampled data. When these errors start to dominate the correlation functions, an erratic behaviour appears without any physics – this phenomenon is known as the Noise Tail. This tail region should be disregarded in an identification of modal parameters and it is possible to estimate the location of the Noise Tail for each structural mode. In this paper, an automated removal of the Noise Tail is introduced and studied and the paper finds that this removal reduces bias and random errors in identification of modal parameters for Operational Modal Analysis.

## 1 Introduction

In Operational Modal Analysis, we can treat the transposed correlation function matrix as free decays, which we can extract modal parameters from [1, 2]. This assumption is based on the properties of ambient vibrations from a linear and ergodic system excited by white Gaussian noise. However, the analytical correlation function matrix is still based on the properties of random vibrations from the system but these exact properties are indeterminable from sample data. This introduces statistical errors when we estimate the correlation function matrix by sample data with a finite time length [3]. Orlowitz and Brandt demonstrated the statistical errors for Operational Modal Analysis in regards to estimation of the modal damping [4]. Giampellegrini derived an expression for the statistical error of the estimated auto-correlation functions for Operational Modal Analysis. Tarpø et al. briefly explained the erratic behaviour in the tail region - the *Noise Tail* - and introduced an algorithm to detect the beginning of the Noise Tail based on the envelope of the modal correlation function [6]. Pridham and Wilson found that the tail region of correlation functions introduces bias in the identification of modal parameters in a numerical study [7]. They found that highly damped modes only need few time lags from the correlation function to get valid results. However, a similar truncation of a correlation function result in errors for modes with low damping and low frequency.

In the literature, we see that the statistical error is related to the finite time length and it creates a bias tail region of the correlation function matrix. Each mode has an individual position of this region in the matrix that will provide the ideal identification of the modal parameters. However, we should not truncate the correlation function matrix too much since this results in errors in the identification of other modes. In this paper, we present an automatic algorithm that reduces the statistical errors while it avoids a heavy truncation of the correlation function matrix. Based on a numerical study, the algorithm reduces bias and random errors of the identification of modal parameters.

## 2 Theory

We are considering a linear system that is excited by white Gaussian noise and we measure a spatial limited system response,  $\mathbf{y}(t)$ . In Operational Modal Analysis, we can calculate the correlation function matrix for the system response and use this matrix as free decays that corresponds to the system parameters, [1, 2].

$$\mathbf{R}_y(\tau) = E [\mathbf{y}(t)\mathbf{y}^T(t + \tau)] \quad (1)$$

We use time averaging instead of ensemble averaging since the response is ergodic. However, the time length must tend toward infinite for this to be true [3].

$$\mathbf{R}_y(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^{T-\tau} \mathbf{y}(t)\mathbf{y}^T(t + \tau) dt \quad (2)$$

where  $T$  is the time length of the measured data. However, properties of random variables cannot be precisely determined from sample data. In reality, a measured system response has a finite time length and this introduces statistical errors since we must estimate the correlation function matrix instead [3].

$$\tilde{\mathbf{R}}_y(\tau) = \frac{1}{T - \tau} \int_0^{T-\tau} \mathbf{y}(t)\mathbf{y}^T(t + \tau) dt, \quad 0 \leq \tau < T \quad (3)$$

This means that the estimated correlation function matrix is a stochastic process due to the statistical errors. The expected values equal the analytic correlation functions (the free decay) and the variance is the statistical errors caused by the finite time length of the recorded signal [3]. By increasing the time length of recording, the variance of the estimated modal correlation functions decreases, see figure 1.

$$\lim_{T \rightarrow \infty} \tilde{\mathbf{R}}_y(\tau) \rightarrow \mathbf{R}_y(\tau) \quad (4)$$

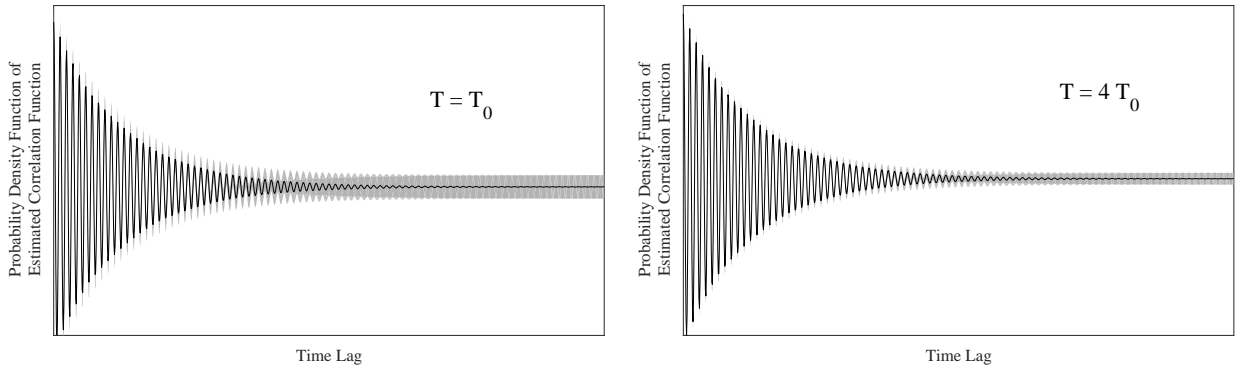


Figure 1: Probability Density Function of the estimated correlation function for an arbitrary system plotted for two different time lengths, the expected value (*black line*) and 2 standard deviations (*gray fill*)

The correlation function matrix has erratic behaviour in the tail region [5, 6], see figure 2. In this part of the correlation functions, the statistical errors dominate and the basic assumption, that the correlation function is a free decay, is invalid. By increasing the time length of the data, the statistical errors decrease, which means that the Noise Tail moves further into the tail region, see figure 3 & 4. This part of the correlation function is biased and its envelope is approximately constant in the Noise Tail as illustrated in figure 4. Therefore the Noise Tail should be excluded from any identification of modal parameters.

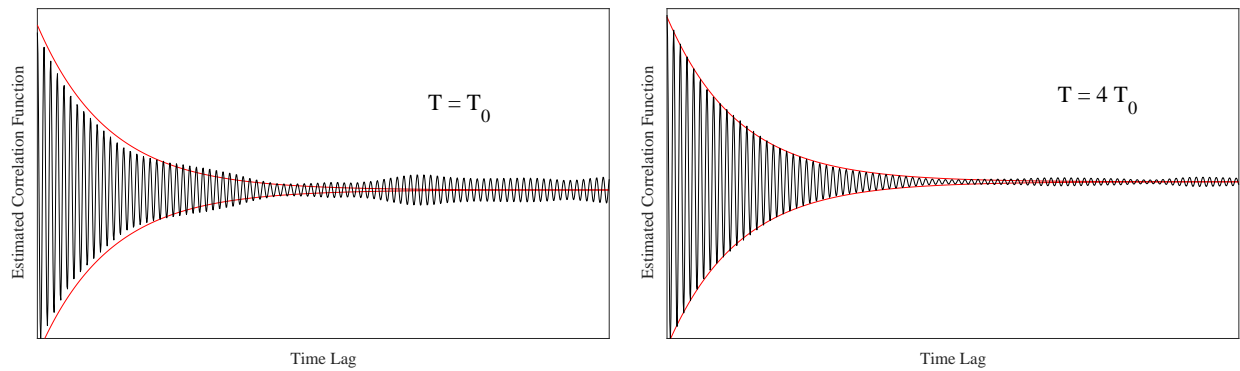


Figure 2: The analytical envelope of the correlation function and the estimated correlation function for an arbitrary system plotted for two different time lengths

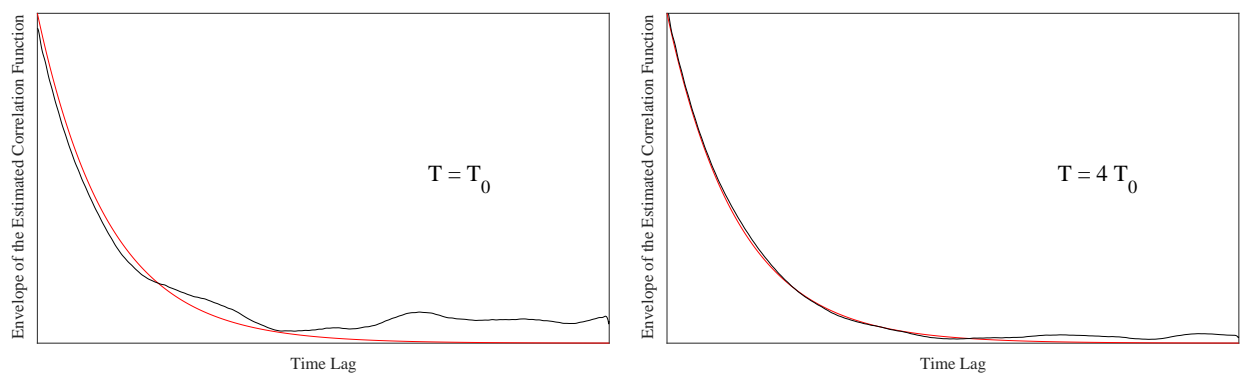


Figure 3: The envelopes of the analytical correlation function and the estimated correlation function for an arbitrary system plotted for two different time lengths

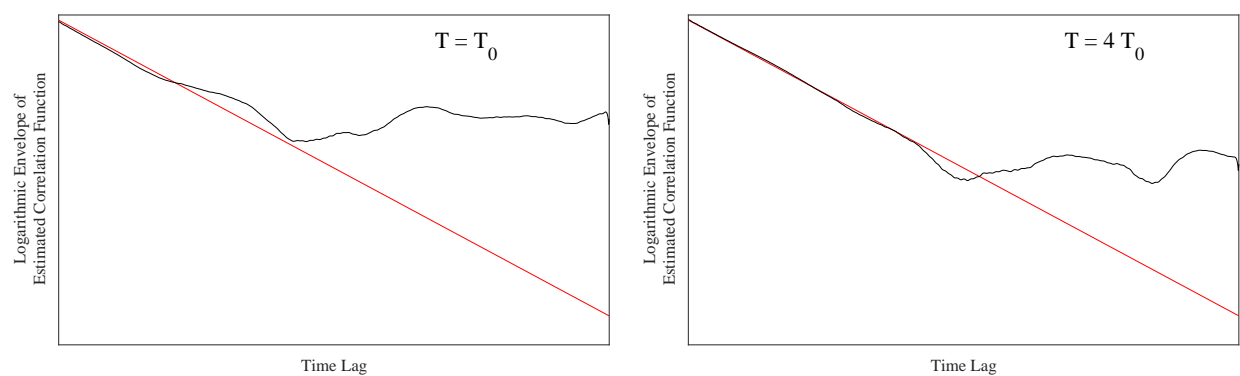


Figure 4: The logarithmic envelopes of the analytical correlation function and the estimated correlation function for an arbitrary system plotted for two different time lengths

For system with  $N$  degrees-of-freedom, we can use the modal decomposition to transform the estimated correlation function to the estimated modal correlation functions, which is the estimated auto-correlation function for the modal coordinates [6, 8].

$$\tilde{\mathbf{R}}_y(\tau) = \sum_{i=1}^N \boldsymbol{\Phi}_i \left[ \tilde{R}_{q_i}(\tau) \right] \boldsymbol{\Phi}_i^H \quad (5)$$

where  $\boldsymbol{\Phi}_i$  is the mode shape for  $i^{th}$  mode. By assuming the statistical errors on the mode shapes are small then this enables us to access the statistical errors on the correlation function matrix as the sum of statistical errors on the modal auto-correlation functions,  $\tilde{R}_{q_i}(\tau)$ . We know that the variance of the estimated auto-correlation function is dependent of the excitation, modal parameters, time length and the time lags [5]. This means that the Noise Tail starts at different points for each modal coordinate. This poses a problem when we need to truncate the correlation function matrix to reduce the bias from the statistical errors. In the case, we truncate the correlation function in order to remove the noise tail from a certain mode then we might have too little left of the correlation function to get proper identification for other modes. We especially have this problem when we deal with combinations of modes with low frequencies and modes with high frequencies and high damping ratios. Here the noise tail begins at a much lower time lag for these modes with high frequency and damping than the modes with lower frequency. However, we need (ideally) a few periods of the modes with low frequency to get good estimation of the modal parameters.

### 3 Algorithm

In this section, we introduce the general idea behind an automated algorithm for detection and reduction of the Noise Tails in the correlation function matrix for Operational Modal Analysis.

We need to decorrelate the correlation function into the modal correlation function, eq. (5). For this we use a combination of principle component analysis [9] and an eigenvalue decomposition to estimate a transformation matrix that decorrelate the correlation function matrix.

We use the algorithm by Tarpø et al. to identify the start of each Noise Tail [6]. This algorithm utilises that the physical part of the correlation function has a sloping logarithmic envelope while the envelope of the Noise Tail is an approximated constant value, see figure 4. It fits two mathematical models to the envelope (a sloping line and a constant value) and the optimal fit indicates the beginning of the Noise Tail.

When we know the position of the Noise Tail for each modal correlation function,  $\tilde{R}_{q_i}(\tau)$ , then we modify the envelope in the Noise Tail region. We change the envelope to mimic that of the physical correlation function by using a the least square fit of a negative exponential from the physical part of the modal correlation function.

Finally, we obtain an modified correlation function matrix where we reduced the statistical errors and we can use this in the identification of modal parameters.

### 4 Case Study

We will use a numerical system with 3 degree-of-freedom to illustrate the algorithm, see the modal parameters in table 1 and the spectral density matrix in figure 5. For this study, we ran 10,000 simulations where we excited the system by white noise. The system response was simulated for 253.77 s (10240 data points) at a sampling frequency of 40.35 Hz (3.1 times the highest natural frequency). We calculated the correlation functions based on the simulated response using the discrete version of eq. (3) with 1025 time lags. In order to simulate the response of the linear system, we utilised the Fourier transformed superposition [8]. For the same simulation, we used the original correlation function matrices and the modified correlation function matrix from the algorithm in the identification of modal parameters. We utilise the Time Domain

Poly-reference technique to identify the modal parameters [10] and the results are plotted in figures 6 & 7 for the original and modified correlation function matrix.

Table 1: Modal parameters of numerical system

	Mode 1	Mode 2	Mode 3
Frequency [Hz]	3.39	10.0	13.0
Damping Ratio [%]	1.49	1.34	1.53

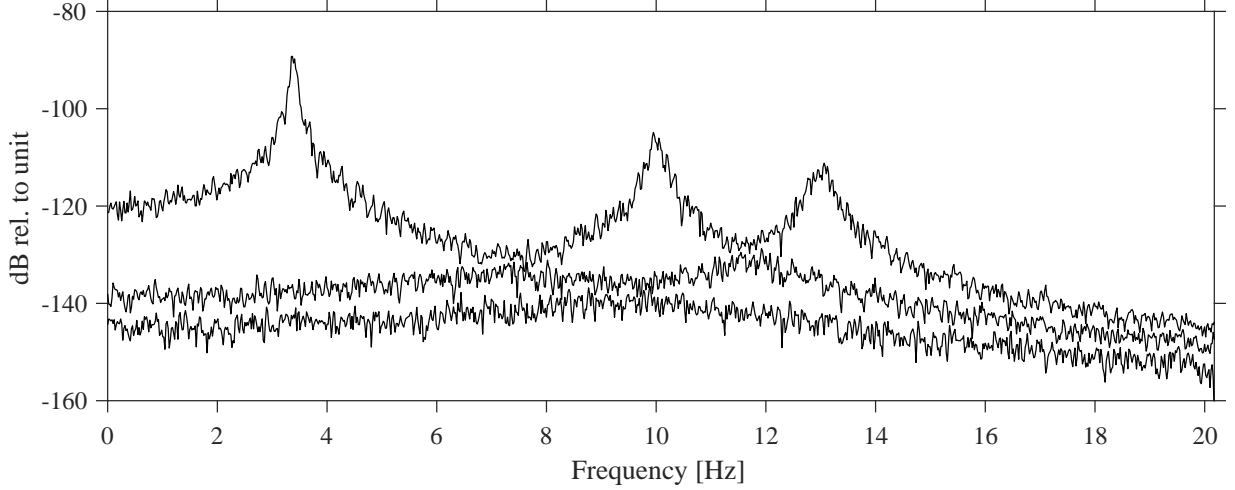


Figure 5: Singular values of the spectral density matrix using the Welch averaging technique [8]

Based on figure 6 & 7, the automated algorithm reduces both the bias and random error in the estimation of modal parameters. Furthermore, the distribution of the modal parameter changes from non-Gaussian to approximately Gaussian distributed.

## 5 Conclusion

We found that the properties of random data cannot be precisely determined from sample data. In other words, the finite time length introduces statistical errors in the correlation function matrix for Operational Modal Analysis and this results in a bias region of the correlation function matrix.

In this paper, we applied an new automatic algorithm for reduction of the statistical errors in the estimated correlation function by removing the noise tail. We found that the algorithm reduces estimation bias while reducing the variance of the modal parameters in this numerical study. For further research, the algorithm will be tested on real data.

## Acknowledgements

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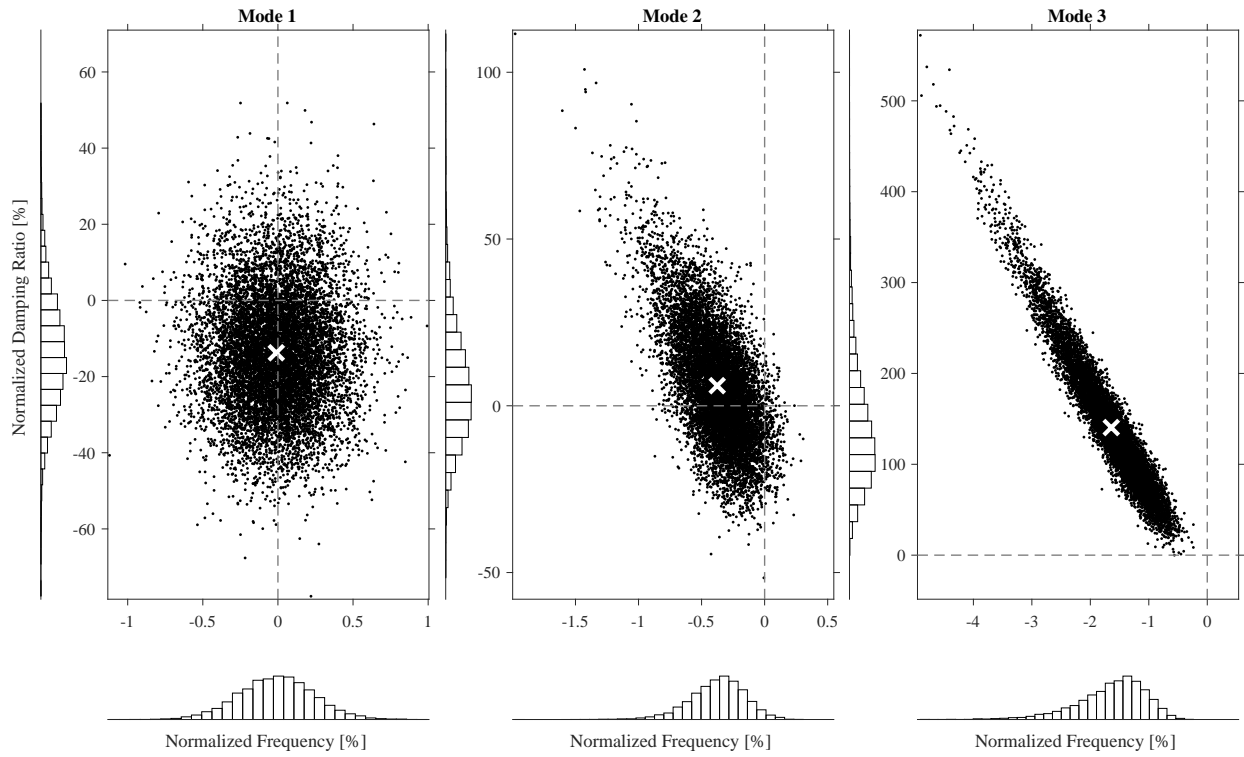


Figure 6: Identification of modal parameters from the original correlation function matrix and their distribution

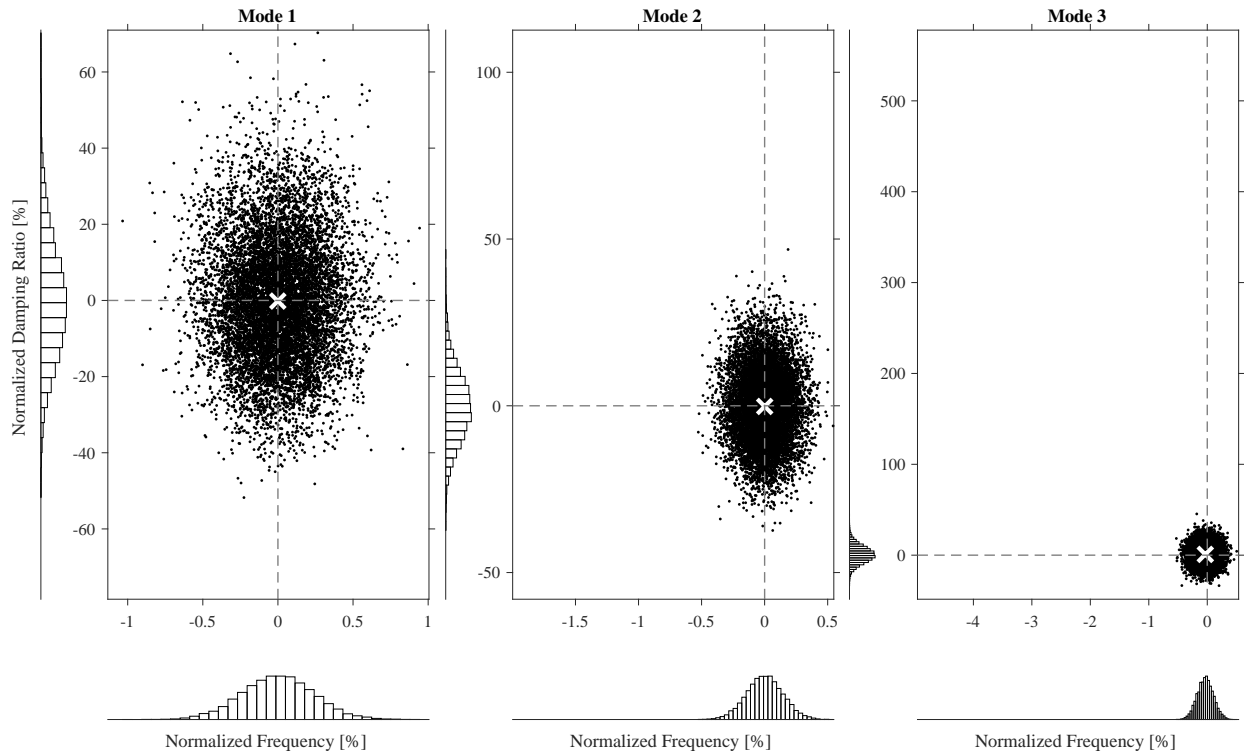


Figure 7: Identification of modal parameters from the modified correlation function matrix and their distribution

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